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A Vizing-type result for semi-total domination

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ABSTRACT

A set of vertices S in a simple isolate-free graph G is a semi-total dominating set of G if it is a dominating set of G and every vertex of S is within distance 2 of another vertex of S. The semi-total domination number of G, denoted by $\gamma_{t2}(G)$, is the minimum cardinality of a semi-total dominating set of G. In this paper, we study semi-total domination of Cartesian products of graphs. Our main result establishes that for any graphs G and H, $\gamma_{t2}(G\Box H) \geq \frac{1}{3}\gamma_{t2}(G)\gamma_{t2}(H)$.

1. Introduction

In this paper we study bounds on a recently introduced domination invariant applied to Cartesian products of graphs. At its core, our work is motivated by the longstanding conjecture of V.G. Vizing [17] on the domination of product graphs, which states that for any graphs G and H, $\gamma(G\Box H) \geq \gamma(G)\gamma(H)$. Here, $\gamma(G)$ is the domination number of G, which is the minimum size of a set D of vertices so that every vertex not in D is adjacent to some vertex in D, and $\Box$ is the Cartesian product of graphs. The breakthrough "double-projection" result of Clark and Suen [5] gave the first Vizing-type bound of $\gamma(G\Box H) \geq \frac{1}{2}\gamma(G)\gamma(H)$. Recently, Brešar [1] improved this bound to $\gamma(G\Box H) \geq \frac{(2\gamma(G)-\rho(G))\gamma(H)}{3}$, where $\rho(G)$ is the two-packing number of G. For more on attempts to solve Vizing's conjecture over more than five decades since it was stated, see the survey [2].

Over the years, due to the unyielding nature of the conjecture, devotees have used offshoots of the domination number to attempt Vizing-type inequalities, in hopes of better understanding the difficulties of the original problem. For example, Brešar, Henning, and Rall [4] defined the paired and rainbow domination numbers, and Henning and Rall [12] conjectured a Vizing-type inequality for total domination. This last conjecture was proved by Ho [7,14], who showed that for any graphs G and H, $\gamma_t(G\Box H) \geq \frac{1}{2}\gamma_t(G)\gamma_t(H)$. In this result, $\gamma_t(G)$ is the total domination number of G, which is the minimum size of a set T of vertices so that every vertex of G is adjacent to some vertex in T. A sharp example was given in [12] and the characterization of pairs of graphs attaining equality is an active problem: see [3] and [15].

Since the difference between a totally dominating set and a dominating set is that every vertex in a totally dominating set must be adjacent to some other vertex in that set, while this rule does not have to hold in a dominating set, we find it instructive to consider Vizing-type inequalities for domination invariants that share properties with both domination and total domination. That is, we want to consider some domination function in between domination and total domination. Such a function, first investigated by Goddard, Henning, and McPillan [6], is the *semi-total domination number of* G, $\gamma_{t2}(G)$, which

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is the minimum size of a set of vertices S in G, so that every vertex of S is of distance at most 2 to some other vertex of S, and every vertex not in S is adjacent to a vertex in S. Although introduced only a few years ago, this function has seen much recent attention, see [8–11,16,18].

Although we cannot prove it, we believe that $\gamma_{t2}(G\,\Box\,H) \geq \frac{1}{2}\gamma_{t2}(G)\gamma_{t2}(H)$ for any graphs G and H. Our result depends on the method of Clark and Suen [5] and requires more careful analysis of semi-total dominating sets. We show that for any graphs G and H, $\gamma_{t2}(G\,\Box\,H) \geq \frac{1}{3}\gamma_{t2}(G)\gamma_{t2}(H)$.

Definitions and Notation. For notations and graph terminologies, we will typically follow [13]. Throughout this paper, all graphs will be considered undirected, simple, connected, and finite. Specifically, let G be a graph with vertex set $V = V(G)$ and edge set $E = E(G)$. Two vertices $v, w \in V$ are neighbors, or adjacent, if $vw \in E$. The *open neighborhood* of $v \in V$, is the set of neighbors of v, denoted by $N_G(v)$, whereas the *closed neighborhood* is $N_G[v] = N_G(v) \cup \{v\}$. The *open neighborhood of* $S \subseteq V$ is the set of all neighbors of vertices in S, denoted by $N_G(S)$, whereas the *closed neighborhood of* S is $N_G[S] = N_G(S) \cup S$. When G is clear from context, we may write $N(S)$ and $N[S]$ instead of $N_G(S)$ and $N_G[S]$, respectively. The distance between two vertices $v, w \in V$ is the length of a shortest (v, w)-path in G, and is denoted by $d_G(v, w)$. The *Cartesian product* of two graphs $G(V_1, E_1)$ and $H(V_2, E_2)$, denoted by $G\,\Box\,H$, is a graph with vertex set $V_1 \times V_2$ and edge set $E(G\,\Box\,H) = \{((u_1, v_1), (u_2, v_2)) : v_1 = v_2 \text{ and } (u_1, u_2) \in E_1, \text{ or } u_1 = u_2 \text{ and } (v_1, v_2) \in E_2\}$.

A subset of vertices $S \subseteq V(G)$ is called a *semi-total dominating set* if $N[S] = V(G)$ and for any vertex $u \in S$, there exists a vertex $v \in S$ so that $d(u, v) \leq 2$. We say that a vertex set S *semi-totally dominates* a vertex set T if S is a semi-total dominating set in the induced subgraph $S \cup T$ of G. The *semi-total domination number of* G, written $\gamma_{t2}(G)$, is the size of a minimum semi-total dominating set of G. A 2-*packing* is a subset of vertices T of G so that every pair of vertices in T is of distance at least 3. The size of a maximum 2-packing of G is called the 2-*packing number*, which is written $\rho(G)$.

We will also make use of the standard notation $[k] = \{1, \ldots, k\}$, and for two vertices u, v, we write $u \sim v$ to indicate that u is adjacent to v.

2. Main results

In this section we provide our main results. We begin by establishing a Vizing's-type result which makes use of the 2-packing number.

Theorem 1. *For any isolate-free graphs G and H,*

$$\gamma_{t2}(G\,\Box\,H) \geq \rho(G)\gamma_{t2}(H).$$

Proof. Without loss of generality, we assume that $\rho(G) = \gamma(G)$ and let $\{v_1, \ldots, v_{\rho(G)}\}$ be a maximum 2-packing of G. Since each vertex from our packing is at distance at least 3 from any other vertex of our packing, we observe that for $i = 1, \ldots, \rho(G)$, the closed neighborhoods $N_G[v_i]$ are pairwise disjoint. Let $\{V_1, \ldots, V_{\rho(G)}\}$ be a partition of $V(G)$ such that $N_G[v_i] \subseteq V_i$, for $1 \leq i \leq \rho(G)$. Let D be a $\gamma_{t2}(G\,\Box\,H)$-set. For $i = 1, \ldots, \rho(G)$, let $D_i = D \cap (V_i \times V(H))$, and let $H_i = \{v_i\} \times V(H)$. Further, let S_i be a minimum set of vertices in $G\,\Box\,H$ that semi-totally dominates H_i and contains as many vertices in H_i as possible. Then, $S_i \subseteq V_i \times V(H)$. Next suppose that S_i contains a vertex x such that x is not in H_i. Then, x is the unique vertex which semi-totally dominates x', for some $x' \in H_i$. Since x' has neighbors $\in H_i$, all of which are dominated by vertices in S_i, if we replace x by x' in S_i, then we see that S_i is still a semi-total dominating set (since x' is at distance at least 2 from a vertex which dominates one of its neighbors). Thus, we have found a set of vertices from $G\,\Box H$ that semi-totally dominates H_i and contains more vertices in H_i than does S_i, a contradiction. Hence, we have $S_i \subseteq H_i$, and so S_i is a semi-total dominating set of the copy of H in $G\,\Box\,H$ induced by the set H_i. Since D_i semi-totally dominates $\{v_i\} \times V(H)$, $|D_i| \geq |S_i|$. Thus,

$$\gamma_{t2}(G\,\Box\,H) \geq \sum_{i=1}^{\rho(G)} |S_i| \geq \sum_{i=1}^{\rho(G)} \gamma_{t2}(H) = \rho(G)\gamma_{t2}(H). \quad \Box$$

Next, we prove a Vizing's type result which relies only on the semi-total domination number. We do this by partitioning minimum semi-total dominating sets into parts that are and are not totally dominating. Notice that for any graph G, if $U = \{u_1, \ldots, u_k\}$ is a minimum semi-total dominating set of G, then U can be separated into two sets, X and Y, where X is the set of vertices of U which are adjacent to at least one other vertex of U, and $Y = U \setminus X$. We call such sets X, *allied* and such sets Y, *free*.

For any graph G, consider the set of minimum semi-total dominating sets of vertices, $\{U_1, \ldots, U_k\}$, and for $1 \leq i \leq k$ let X_i and Y_i be partitions of U_i into allied and free sets, respectively. We call U_i so that $|X_i|$ is of maximum size for $1 \leq i \leq k$ a *maximum allied semi-total dominating set* of G, the partition $\{X_i, Y_i\}$ a *maximum allied partition* of G, the set X_i a *maximum allied set* of G, and the set Y_i a *minimum free set* of G.

For any maximum allied partition of G, $\{X, Y\}$, let $x(G) = |X|$ and $y(G) = |Y|$.

Theorem 2. *For any isolate-free graphs G and H,*

$$\gamma_{t2}(G\,\Box\,H) \geq \frac{1}{3}\gamma_{t2}(G)\gamma_{t2}(H)$$

Proof. Let D be a minimum semi-total dominating set of $G\,\square\, H$. Let $k = \gamma_{t2}(G)$ and $U = \{u_1, \ldots, u_k\}$ be a maximum allied semi-total dominating set of G with maximum allied partition $\{X, Y\}$. Suppose $X = \{u_1, \ldots, u_\ell\}$ and $Y = \{u_{\ell+1}, \ldots, u_{\ell+m}\}$.

Form a partition $\{\pi_1, \ldots, \pi_k\}$ of $V(G)$ where $\pi_i \subseteq N(u_i)$ and $x \in \pi_i$ implies x is adjacent to u_i for $1 \le i \le \ell$, $\pi_j \subseteq N[u_j]$ and $x \in \pi_j$ implies x is adjacent to or equal to u_j for $\ell + 1 \le j \le \ell + m = k$. Furthermore, we define this partition to have the property that if $u_i \in X$ and $u_j \in Y$ so that $d(u_i, u_j) = 2$, then $N(u_i) \cap N(u_j) \cap \pi_j = \emptyset$. That is, for any vertex u_j of Y which is of distance 2 to some vertex of X, there exists a vertex u which is adjacent to u_j and to a vertex of X, and u belongs to π_i for some $i \in [\ell]$. To explain, if a vertex u is both a neighbor of an element in X and an element in Y, then when selecting our partition, we place u in the part containing the element of X, not Y. This choice is made to minimize the sizes of π_j for $\ell + 1 \le j \le k$.

Let $D_i = (\pi_i \times V(H)) \cap D$. Let $P_i = \{v : (u, v) \in D_i$ for some $u \in \pi_i\}$, which are the projections of D_i onto H. We call vertices of $V(H)$ *missing*, if they are not dominated from P_i and write $M_i = V(H) - N_H[P_i]$. Vertices of P_i which are of distance at most 2 to some other vertex of P_i or M_i we call *covered* and write $Q_i = \{v \in P_i : \exists w \in P_i \cup M_i$ such that $0 < d(v, w) \le 2\}$. Vertices of P_i of distance at least 3 to other vertices of P_i or M_i we call *uncovered* and write $R_i = \{v \in P_i : \forall w \in (P_i \cup M_i)\setminus\{v\}, d(v, w) \ge 3\}$.

For $v \in V(H)$, let

$$D^v = D \cap (V(G) \times \{v\}) = \{(u, v) \in D : u \in V(G)\}$$

and C be a subset of $\{1, \ldots, k\} \times V(H)$ given by

$$C = \{(i, v) : \pi_i \times \{v\} \subseteq N_{G\square H}(D^v) \text{ or } v \in R_i\}.$$

Let $N = |C|$. We will bound N from above by considering the following.

$$\mathcal{L}_i = \{(i, v) \in C : v \in V(H)\},$$

$$\mathcal{R}^v = \{(i, v) \in C : 1 \le i \le k\}.$$

These definitions are well-known as they appeared in the seminal work [5], nonetheless, we would like to remind the reader of their interpretation. The set C is a double indexing set, which indicates where you have cells that are either horizontally dominated or dominated by vertices of R_i. A cell is just a copy of π_i for some i, at some height $v \in V(H)$. We represent G along the horizontal axis of the Cartesian product and H along the vertical. Thus, horizontally dominated cells are precisely, $\pi_i \times \{v\}$ which is contained in $N_{G\square H}(D^v)$. Now, L_i are elements of C with a fixed i and R^v are elements of C along a fixed v.

Since counting vertices vertically and horizontally produces the same amount, we have

$$N = \sum_{i=1}^{k} |\mathcal{L}_i| = \sum_{v\in V(H)} |\mathcal{R}^v|.$$

Notice that if $v \in M_i$, then the vertices in $\pi_i \times \{v\}$ which are not in D^v must be adjacent to the vertices in D^v since D is a semi-total dominating set of $G\,\square\, H$. Furthermore, the vertices of R_i are counted in $\mathcal{L}_i$. This means that $|\mathcal{L}_i| \ge |M_i| + |R_i|$. Hence we obtain the following lower bound for N,

$$N \ge \sum_{i=1}^{k} (|M_i| + |R_i|) \tag{1}$$

To find an upper bound on the above quantity, we bound the size of $\mathcal{R}^v$.

Claim 1. *For any $v \in V(H)$, $|\mathcal{R}^v| \le 2|D^v|$.*

Proof. Suppose $|\mathcal{R}^v| > 2|D^v|$ for some $v \in V(H)$. For $(i, v) \in \mathcal{R}^v$, by definition, $\pi_i \times \{v\} \subseteq N_{G\square H}(D^v)$ or $v \in R_i$.

In what follows, we construct a semi-total dominating set T of G.

In the **first case**, if $\pi_i \times \{v\} \subseteq N_{G\square H}(D^v)$, we note that if some vertex $x \in \pi_i$, then x is adjacent to vertices in B^v where B^v is the projection of D^v onto G.

Subcase 1. *Suppose* $u \in B^v$. If $u \in \pi_i$ such that $(i, v) \notin \mathcal{R}^v$, $u \ne u_i$ and $1 \le i \le \ell + m$, then $u \in N(u_i)$. If $u \in \pi_i$ such that $(i, v) \in \mathcal{R}^v$, then there exists $u' \in B^v$ such that $u \in N(u')$. If $u \in \pi_i$ such that $(i, v) \notin \mathcal{R}^v$, $u = u_i$ for some $\ell + 1 \le i \le \ell + m$, then notice that we can find a vertex x_i which is a neighbor of u_i in π_i. Note that x_i need not be a member of B^v, but simply a neighbor of u_i. Select one such vertex x_i for every such u, and let A be the set of these vertices x_i. Thus, $B^v \subseteq T$, $A = \{u_i : (i, v) \notin \mathcal{R}^v, u_i \notin B^v, 1 \le i \le \ell + m\} \subseteq T$, and $\{x_i : (i, v) \notin \mathcal{R}^v, x_i \sim u$ for some $u \in U \cap B^v\} \subseteq T$.

Subcase 2. *Suppose* $u \in \{u_i : (i, v) \notin \mathcal{R}^v, 1 \le i \le \ell\}$. If $u \in \pi_j$ such that $(j, v) \notin \mathcal{R}^v$, then $u \in N(u_j)$. If $u \in \pi_j$ such that $(j, v) \in \mathcal{R}^v$, then there exists $u' \in B^v$ such that $u \in N(u')$. Thus, in this subcase, u is adjacent either to a vertex of B^v or a vertex u_j. There are no new vertices that need to be added to T.

Subcase 3. *Suppose* $u \in \{u_i : (i, v) \notin \mathcal{R}^v, \ell + 1 \le i \le \ell + m\}$. Suppose u is of distance 2 to some vertex $u_j \in X$. By the definition of the partition, there exists some vertex w adjacent to u and u_j, so that $w \in \pi_{j'}$ for some $j' \in [\ell]$. If $(j', v) \in \mathcal{R}^v$, then there exists $u' \in B^v$ so that $u' \sim w \sim u$, which means that u is of distance at least 2 to some vertex of B^v. Since T contains B^v, these vertices are already distance 2 from another vertex in T.

We are left to consider the case when u is of distance at least 3 to any vertex of X. Since U is a minimum semi-total dominating set of G, there exists some vertex $u_j \in Y$, so that $d(u, u_j) = 2$. If $(j, v) \notin \mathcal{R}^v$, these vertices are already in T so no action needs to be taken.

If $(j, v) \in \mathcal{R}^v$, then there exists some vertex $u' \in B^v$ so that $u' \sim u_j$. We will select u_j and place it in T to make T a semi-total dominating set of G. Notice that in this case, the number of such vertices u_j is at most equal to $|D^v|$. Let S be the set of such vertices u_j, which are of distance 2 to a vertex $u \in Y$ and at least of distance 3 to any vertex of X. Then S will be a subset of the set T. This finishes Subcase 3.

In the **second case**, if $v \in R_i$, then since D is a semi-total dominating set, there is some vertex $(u, v) \in (\pi_i \times \{v\}) \cap D^v$ and $(w, v) \in (\pi_j \times \{v\}) \cap D^v$, for some $j \in [k]$, so that (u, v) is at most distance 2 from (w, v).

Putting these cases together, we have the following disjoint union of sets:

$$T = B^v \cup \{u_i : (i, v) \notin \mathcal{R}^v, 1 \le i \le \ell\} \cup \{u_i : (i, v) \notin \mathcal{R}^v, u_i \notin B^v, \ell + 1 \le i \le \ell + m\} \cup A \cup S \tag{2}$$

To show T is a semi-total dominating set of G, it is enough to show that T is a dominating set, since we showed in each subcase of the first case, and in the second case, that every vertex of T is of distance at most 2 to some other vertex of T. If a vertex u is contained in π_i for $(i, v) \in \mathcal{R}^v$, then u is dominated by some vertex of B^v. If $(i, v) \notin \mathcal{R}^v$, then u is dominated either by $\{u_i : (i, v) \notin \mathcal{R}^v, 1 \le i \le \ell\}$, or $\{u_i : (i, v) \notin \mathcal{R}^v, u_i \notin B^v, \ell + 1 \le i \le \ell + m\}$, or A.

Furthermore,

$$|T| = |B^v| + (\gamma_{t2}(G) - |\mathcal{R}^v| + |S|) = 2|D^v| + (\gamma_{t2}(G) - |\mathcal{R}^v|) < \gamma_{t2}(G)$$

which is a contradiction. □

Thus, by Claim 1,

$$N = \sum_{v \in V(H)} |\mathcal{R}^v| \le \sum_{v \in V(H)} 2|D^v| = 2|D| \tag{3}$$

We now show a semi-total dominating set of H in terms of M_i.

Claim 2. *For any $i \in [k]$, there exists a set X_i of at most $|R_i| - 1$ vertices of $V(H)$ so that $M_i \cup P_i \cup X_i$ is a semi-total dominating set of H.*

Proof. We first observe that $P_i \cup M_i$ is a dominating set of H with the additional property that the vertices of M_i dominate only themselves, not their neighbors. Thus, every vertex $x \in R_i$ must be either of distance 3 to some vertex $y \in R_i$ or every vertex of distance 2 from x is a vertex of M_i. This holds since otherwise some vertex of distance 2 from x is not dominated by $P_i \cup M_i$. Furthermore, if $x \in R_i$ which is of distance 3 to some vertex $y \in R_i$, then we may select one vertex z on a path from x to y such that z is of distance at most 2 to both x and y.

We now construct a semi-total dominating set of H, T_i, by including the vertices of M_i, the vertices of P_i and vertices X_i which are of distance at most 2 to two vertices of R_i which are themselves of distance three to each other. The minimum number of such vertices is at most $|R_i| - 1$, which can be easily verified by induction on $|R_i|$, and the result follows. □

By Claim 2, for each i, we can construct a semi-total dominating set of H, $T_i = M_i \cup R_i \cup Q_i \cup X_i$. This gives $|M_i| + |R_i| \ge \gamma_{t2}(H) - |X_i| - |Q_i|$. However, note that $X_i \cap Q_i = \emptyset$ and $|X_i| + |Q_i| \le |P_i|$. This implies that $|M_i| + |R_i| \ge \gamma_{t2}(H) - |P_i|$. Thus, we have

$$\sum_{i=1}^{k} \Big(|M_i| + |R_i| \Big) \ge \sum_{i=1}^{k} \Big(\gamma_{t2}(H) - |P_i| \Big) = \gamma_{t2}(G)\gamma_{t2}(H) - |D| \tag{4}$$

Combining Eqs. (1), (3), and (4) we obtain

$$|D| \ge \frac{1}{3}\gamma_{t2}(G)\gamma_{t2}(H) \quad \square$$

3. Conclusion

In this paper we have proven two Vizing-type results on the semi-total domination number. Our main result, in Theorem 2, shows that for isolate-free graphs G and H, we have the inequality $\gamma_{t2}(G \square H) \ge \frac{1}{3}\gamma_{t2}(G)\gamma_{t2}(H)$. However, we do not believe this bound is sharp, and conjecture a stronger result.

Conjecture 1. *For any isolate-free graphs G and H,*

$$\gamma_{t2}(G \square H) \ge \frac{1}{2}\gamma_{t2}(G)\gamma_{t2}(H).$$

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